# Characterization and modelling of broad-band GHz-field A/D acquisition channels by means of the Discrete-Time Convolution Model behavioural approach

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*Abstract-* The Discrete-Time Convolution Model behavioural approach is proposed for the characterization/empirical modelling of high-performance broad-band A/D channels, designed for the acquisition of signals approaching the microwave field. Thanks to its inherent capability of separately describing all the non-idealities of different nature within the channel, and in particular the non-linear dynamic effects, the method is particularly suitable for the A/D channels of modern high-frequency instrumentation and RF receivers. Experimental results are provided, which validate the analytical formulation of the approach and the related measurement procedures/set-ups for model parameter experimental extraction, for a state-of-the-art, 1-GHz analog bandwidth 1-GSa/s A/D acquisition channel for satellite radar applications.

## I. Introduction

   High-sampling frequency, broad-band analog-to-digital (A/D) acquisition channels represent fundamental components not only in modern digital instrumentation, but also for an ever-growing family of terrestrial, mobile, airborne and satellite communication systems, including radar and radiometry applications. In fact, a common aspect among such systems can be identified in the trend of designing RF receiving architectures, in which the A/D conversion sections are nearer and nearer to the antenna subsystem, both in terms of reduction of down-conversion analog blocks employed, and in terms of operative frequencies involved. The most important reasons for this are the improved linearity of the receiver, reduced noise, higher flexibility and reconfigurability of the systems.

   The experimental characterization and empirical modelling of an A/D channel (or a stand-alone A/D Converter (ADC)) designed for broad-band input signals are challenging tasks, since the approach followed must take into account and suitably describe the *non-linear dynamic* effects, which become important in the behaviour of the channel in the presence of high-frequency operating conditions. Most of the classical methods offered by the literature, involving a quasi-static interpretation of the non-linear contribution to the response of the A/D channel and the use of slow-varying test signals, are usually not adequate enough to this aim. On the other end, static and dynamic figures of merit suggested by the standards (e.g., [1,2]), although providing useful information about the distortion performance of the channel under simple, reference operating conditions, are not strictly speaking a model, and fail to predict the behaviour of the system when a generic signal regime (in terms of amplitude levels and spectral complexity) is applied to its input. Moreover, they can not separately describe the contribution of the dynamic non-linearities to the output perturbation, with respect to that due to static non-linear effects. This feature, if available, can be instead very important when the obtained model is exploited for the compensation of the non-idealities in the A/D channel behaviour.

   The Discrete-Time Convolution Model (DTCM) has been proposed in the past [3,4] for the accurate, separate characterization of all the non-idealities of different nature within A/D channels. This behavioural, technology-independent approach, which is based on a modification [5] of the conventional Volterra series [6,7], is capable of explicitly describing the dynamic non-linearities within the A/D channel by means of a suitable set of parameters [8]. In this paper, the general-purpose DTCM methodology is proposed for high-performance, broad-band A/D channels, and experimental results are provided in the framework of satellite radar applications. To this aim, while the original analytical formulation of the model can be applied without the need for substantial modifications, the experimental procedures and related measurement set-ups devoted to the empirical extraction of DTCM parameters have been completely re-designed with respect to those validated for applications at lower frequencies (see [9]).

## II. The DTCM approach for broad-band A/D channels

### A. General formulation of the non-linear dynamic characterization/modelling methodology

   In Figure 1 the functional representation for the DTCM model is shown. The actual M-bit A/D channel with input signal $s_I(t)$ is equivalently described by means of an *ideal* ADC that quantizes and converts to digital (at

each sampling instant $t_k$ provided by an external time-base clock) the output $y(t)$ of a non-linear dynamic system (gray-filled shape in the figure), whose purpose is to take into account all the non-linear effects (both static and dynamic) in the overall channel behaviour. This non-linear system is supposed to be characterized by a memory time duration which is *short* when compared to the minimum period of the typical signals applied to the A/D channel, in order to analytically describe its input/output relationship by means of the modified Volterra series expansion truncated to the first-order integral contribution [5].
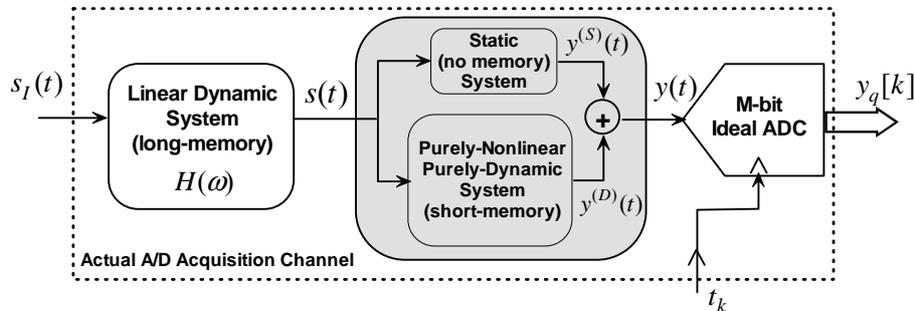


Figure 1. General functional representation of the Discrete-Time Convolution Model for entire A/D acquisition channels or stand-alone ADCs.

According to this "short-memory" hypothesis, in Eq. (1) the instantaneous value of the output $y(t)$ is given as the superposition of the zero- and first-order series contributions ($y^{(S)}(t)$ and $y^{(D)}(t)$, respectively):

$$y(t) = y^{(S)}(t) + y^{(D)}(t) \quad with \quad y^{(S)}(t) = z_0[s(t)] \quad ; \quad y^{(D)}(t) = \int_{-T_A}^{T_B} w[s(t),\tau][s(t-\tau)-s(t)]d\tau \quad (1)$$

The single-fold integral in Eq. (1), which is computed over the (short) memory time interval $[-T_A, T_B]$ and includes a first-order modified kernel $w$ non-linearly controlled by the input $s(t)$ of the system, is a *purely-dynamic* term and describes the contribution to $y(t)$ due to the dynamic non-linearities affecting the A/D channel behaviour. By means of a straightforward discretization with step $\Delta\tau$ of the memory time, and a polynomial expansion in $s$ of the first-order kernel $w[s(t),\tau]$, the contribution $y^{(D)}(t)$ can be rewritten as follows:

$$y^{(D)}(t) \cong \sum_{\substack{p=-P_A \\ p\neq0}}^{P_B} [s(t-p\Delta\tau)-s(t)]\sum_{n=1}^{N}\beta_{pn}s^n(t) \quad (2)$$

In Eq. (2) the matrix of parameters $\boldsymbol{\beta} = \{\beta_{pn}\}$ is thus defined, which explicitly and separately characterizes the non-linear dynamic effects perturbing the behaviour of the A/D channel. As far as the zero-order term $y^{(S)}(t)$ of the modified Volterra series is concerned, it is easy to show that the memoryless (i.e., algebraic) non-linear function $z_0$ in Eq. (1) represents the static response of the A/D channel.

Finally, the linear dynamic system representing the DTCM model front-end in Fig. 1 describes the long-memory (although practically linear) effects typical of the A/D channel *i)* input signal conditioning circuitry, and *ii)* access network to the actual A/D conversion section (e.g., coaxial connector, impedance matching, single-ended to balanced transition, level attenuation, filtering, transmission lines, etc.). The presence of this behavioural block allows for the de-embedding of the non-linear system that follows from long-memory dynamics, thus it makes possible the introduction of the short-memory hypothesis in the modelling of channel non-linearities.

## B. Measurement techniques for the DTCM characterization of broad-band high-frequency A/D channels

The original experimental procedure [9] devoted to the extraction of the DTCM parameters $\boldsymbol{\beta}$ made use of a *reference* A/D channel in parallel with the DUT in order to measure, with a much higher degree of accuracy with respect to the performance of the latter, the input signals generated during a set of frequency- and amplitude-varying sinusoidal tests. However, when dealing with state-of-the-art, broad-band A/D channels with input signals getting near to the field of microwaves, such a strategy can be non-feasible from a practical standpoint,

since a reliable reference channel (better linearity, same or better analog bandwidth) could be hardly available.

In this work, a new experimental methodology has been designed and applied, which makes use of a reference A/D channel that is nominally *identical* to the DUT (i.e., basically a second sample of the DUT is exploited: same technology and architecture, same performance), but that operates under *linear* conditions during the tests.
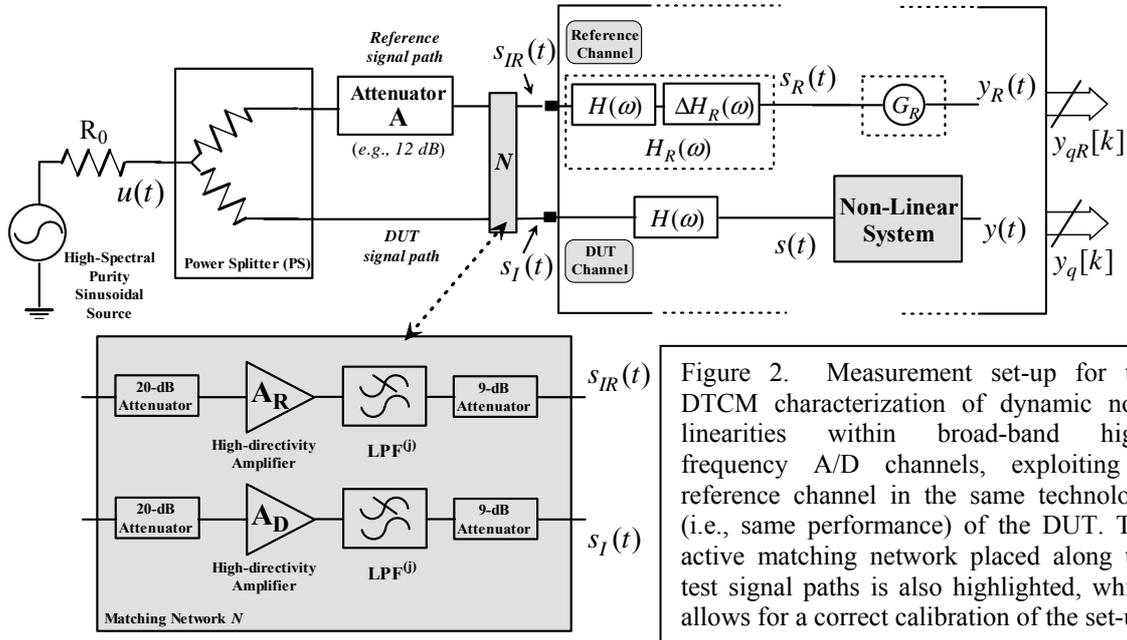


Figure 2. Measurement set-up for the DTCM characterization of dynamic non-linearities within broad-band high-frequency A/D channels, exploiting a reference channel in the same technology (i.e., same performance) of the DUT. The active matching network placed along the test signal paths is also highlighted, which allows for a correct calibration of the set-up.

Figure 2 describes the laboratory set-up which has been developed for the experimental extraction of DTCM parameters **β** for broad-band A/D channels. A high-performance resistive power splitter ([DC – 10 GHz]) is employed to equally distribute towards two paths the signal $u(t)$ generated by a highly pure microwave sinusoidal source (Agilent E8257D-520 with harmonic filtering option). The first signal path leads to the A/D channel under test (DUT), while the second path is loaded with the input of the reference A/D channel. The network $N$ is only aimed at impedance active matching and can be ignored during the present description. Thus, the only architectural asymmetry between the two signal paths is the presence of a precision attenuator A along the reference one: while the level of the source is accurately set (for each test frequency) in order to operate the DUT at full-scale (i.e., the output records $y_q[k]$ cover all the possible code bins), the value of attenuation for A can be chosen for the given DUT architecture in order to maintain the harmonics at the output of the reference channel slightly under the quantization noise floor. As a result, the reference channel can be described as shown in the figure by a *linearized* DTCM representation, in which the purely non-linear dynamic contribution $y_R^{(D)}(t)$ is neglected, and the static response $z_{0,R}$ simply reduces to its linearization $G_R$ around the origin. Obviously, the input linear network with transfer function $H_R(\omega)$ is not affected by the linearization.

The laboratory set-up of Figure 2 can be successfully exploited in order to perform the DTCM characterization of the DUT dynamic non-linearities. More precisely, it allows to estimate the *linear* dynamic relationship between the non-directly accessible signal $s(t)$ (i.e., the input of the non-linear dynamic system in Fig. 1), and the output of the reference channel, which is instead experimentally available for any given sinusoidal test excitation $u(t)$ with frequency $\omega_0$ sweeping throughout the analog bandwidth of the A/D channel. In fact, the following equation can be easily written:

$$d \cdot S(\omega)e^{-j\omega\Delta t_A} = \frac{C(\omega)}{A_0} \cdot Y_R(\omega) \qquad (3)$$

where $[S(\omega), Y_R(\omega)]$ are the complex representation of signals $s(t)$ and $y_R(t)$, respectively, $A_0$ is the nominal value of attenuation of A, and $C(\omega)$ is a frequency-dependent coefficient that can be estimated by means of an accurate calibration of the set-up. In particular, by moving the attenuator A from the reference signal path to the input of the power splitter, *both* channels can be considered as operating under linear conditions. It can be shown

that the coefficient $C(\omega)$ in Eq. (3), which describes the residual asymmetry between the two channels due to the non-idealities of the access path components and, in particular, the unbalance of the power splitter and the factor deviation $\Delta H_R(\omega)$ of reference channel front-end network transfer function $H_R(\omega)$ with respect to $H(\omega)$, is thus empirically obtained at a given test frequency $\omega_0$ as

$$C(\omega_0) = \frac{Y(\omega_0)}{Y_R(\omega_0)} \tag{4}$$

where the complex representations $[Y(\omega_0), Y_R(\omega_0)]$ are estimated by means of a four-parameter fit to sine-wave [2] of output records $[y_q[k], y_{qR}[k]]$, respectively.

The experimental extraction of the DTCM parameters $\boldsymbol{\beta}$ is therefore carried out through the set-up of Fig. 2 by generating a set of $J$ test signals $u_j(t)$, from low frequency up to the bounds of the A/D channel analog bandwidth, considering for each test a sub-set of $M$ sampling instants $t_m$ at which the output of the DUT is available and solving, by means of least-square algorithms, the following over-determined system of linear equations in the $(P_A + P_B)N \ll J \cdot M$ unknowns $\boldsymbol{\beta} = \{\beta_{pn}\}$ :

$$\sum_{\substack{p=-P_A \\ p \neq 0}}^{P_B} \left[ s_j(t_m - p\Delta\tau) - s_j(t_m) \right] \sum_{n=1}^{N} \beta_{pn} s_j^n(t_m) = y_j^{(D)}(t_m) =$$

$$= y_j(t_m) - z_0 \left[ s_j(t_m) \right] \qquad (j = 1,\ldots,J)(m = 1,\ldots,M) \tag{5}$$

The estimates of the $j$-th signal $s_j$ required in system (5) are obtained by the continuous-time transformation of the right-end side of Eq. (3). It is worth noticing that Equation (3) provides, rigorously speaking, the estimate $\hat{s}_j(t) = d \cdot s_j(t - \Delta t_A)$, since after the calibration procedure a frequency-independent coefficient $d$ (near to the unity) and a time delay $\Delta t_A$ (introduced by the precision attenuator A, for which the linear non-distortion hypothesis has been assumed) are still unknown. While the solution of (5) shows a negligible sensitivity to the factor $d$, the estimate for $\Delta t_A$ is obtained by means of an optimization procedure, based on the iterative solution of (5) with the goal of minimizing a suitable error function. The static characteristic $z_0$ of the A/D channel, which is needed to build the known vector term in Eq. (5), can be estimated by means of one of the many methods available in the literature. In this work, a simple polynomial expansion has been adopted, since it has been found to provide more accurate predictions with respect to other approaches devoted to the representation of static non-linearities, when the distortion at the output of the A/D channel is considered in the presence of near full-scale signals at the input. In addition, by adopting the polynomial expansion, the associated coefficients can be operatively made part of the unknowns in system (5): as a result, in the practice the A/D channel static response is characterized at the same time (i.e., by solving (5)) as the dynamic non-linearities.

Thus, the architecture of Fig. 2 is capable of providing the experimental information needed for the complete characterization/modelling of the A/D channel dynamic non-linearities according to the DTCM approach, without the need for a higher-performance auxiliary channel, and therefore it allows for the successful application of the DTCM methodology to broad-band, GHz-field A/D channels.

The quality of the calibration procedure is clearly a key factor in order to obtain an accurate extraction of the DTCM parameters. In particular, the frequency response of all the set-up microwave components should not appreciably change when switching from the calibration configuration (i.e., attenuator at the input of the power splitter) to the operative one (i.e., attenuator on the reference path). Since the two A/D channels cannot usually provide at their input the high-performance 50-$\Omega$ impedance matching typical of precision microwave laboratory components, this condition is not always guaranteed. For this reason, the active matching network $N$ shown in Fig. 2 has been designed and implemented. Each signal path is provided with a high-directivity, high-linearity amplifier (capable of strongly rejecting the waves reflected at the A/D channel inputs), driven by an attenuator of high value (20 dB). The amplifiers are followed by attenuators, in order to correctly operate, and low-pass filters are employed, to suppress the (low) distortion introduced by the active elements. The entire analog bandwidth of the DUT can be covered by exploiting a sufficient number of filters. As a result, both paths are loaded with a nearly-ideal 50-$\Omega$ impedance, so that the main condition required for a very accurate calibration of the set-up is fulfilled.

### III. Experimental results

In order to provide a validation example of the DTCM approach and related experimental procedures/set-ups, an A/D acquisition channel will be considered in the following, whose conversion device is a 8-bit, 1-GSa/s low-power differential input ADC qualified for space applications. The A/D channel front-end analog circuits are a coaxial connector, a balun-like AC-coupling transition from single-ended to differential and a matching network to implement the 50-Ω input impedance typical of RF systems. The analog bandwidth of the channel achieves 1 GHz. The DUT has been fully characterized by means of the DTCM improved methodology and the obtained model implemented in the framework of a CAD software package for the non-linear simulations of microwave circuits and systems.
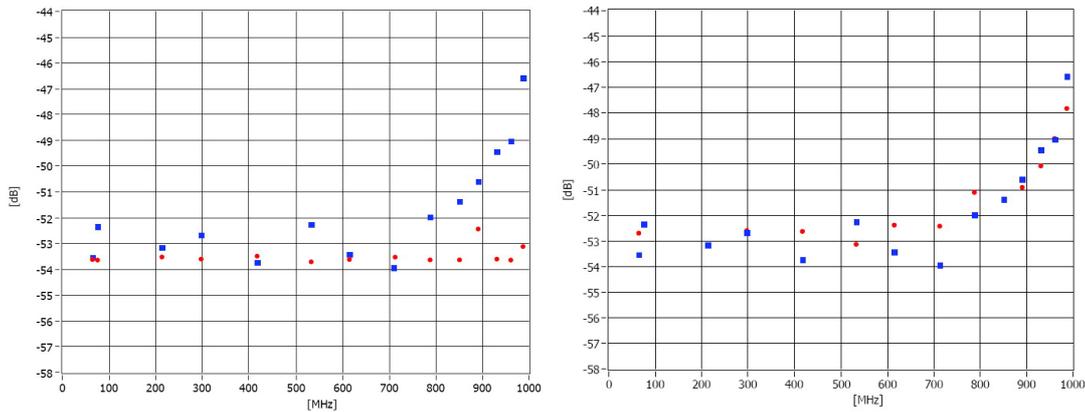


Figure 3. Total Harmonic Distortion of the DUT measured (blue squares) compared to DTCM predictions (red dots). Left: only the static non-linearities are included in the simulation. Right: the DTCM model is fully exploited, including the contribution of dynamic non-linearities.

Several validation tests have been performed by comparing the predictions provided by the DTCM model of the DUT and the corresponding measurement data. In Fig. 3 the THD of the A/D channel measured in the frequency band [70 MHz – 1 GHz] is shown (blue squares). As can be appreciated, the level of THD is very low (the ADC employed is a high-performance device in terms of linearity) and practically flat up to 700 MHz, showing that the non-linear effects are mainly of static nature below this frequency. However, the THD definitely grows beyond 700 MHz due to the increasing contribution of the dynamic non-linearities, with a quite strong reduction of the linearity performance of the channel. In Fig. 3 (left) the prediction provided by the DTCM model is also shown (red dots), but by *not* activating, during the simulation, the dynamic non-linear system of Fig. 1 (i.e., only the static non-linear effects modelled by the DTCM implementation give contribution to the simulated behaviour of the A/D channel): the predictions are very accurate up to 700 MHz, showing that the static response of the DUT has been correctly characterized, but they remain flat at higher frequencies (as expected), failing to reproduce the experimental data. In the right graph of Fig. 3 the predictions obtained by exploiting the *full* DTCM implementation are instead shown. Thanks to the activation of the non-linear dynamic contribution $y^{(D)}(t)$, the dynamic non-linearities are correctly taken into account and the predictions are now very accurate up to 1 GHz.

Figure 4 shows the statistical distribution of the deviation (unit: LSB) between measured and DTCM-predicted samples at the output of the A/D channel, when a full-scale sinusoidal signal is applied at the input. At low frequency (left, $f_0$ = 157 MHz) the simulation obtained by activating only the DTCM static non-linear contribution is accurate (more than 70% of predicted samples are correct, with all the remaining within 1 LSB): the static non-linearities are the dominant effect at this frequency, and the use of the full DTCM model does not either improve or degrade the predictions. At high frequency, instead, (right, $f_0$ = 989 MHz) the use of the full DTCM model is necessary in order to achieve the best prediction performance, since dynamic non-linearities now strongly perturb the behaviour of the A/D channel.

Figure 5 is an example of the far most critical comparison between experimental data and model predictions. For a full-scale sinusoidal input at $f_0$ = 931 MHz, the FTTs on the measured (left) and DTCM-predicted (right) record at the output of the channel are shown. Although such a test does not involve, somehow, an averaging of the spectral content, as in the case of the THD, but, quite the opposite, since consisting in the comparison harmonic-by-harmonic, can point out not only subtle inaccuracies of the model, but also the effect of perturbing phenomena, which are not due to the channel behaviour (such as the non-idealities of the time-base), and thus not taken into account by the model, Fig. 5 demonstrates very good DTCM-predictions at II and III harmonics in the output spectrum, even when the A/D channel is operated at the upper-bound of its analog bandwidth.
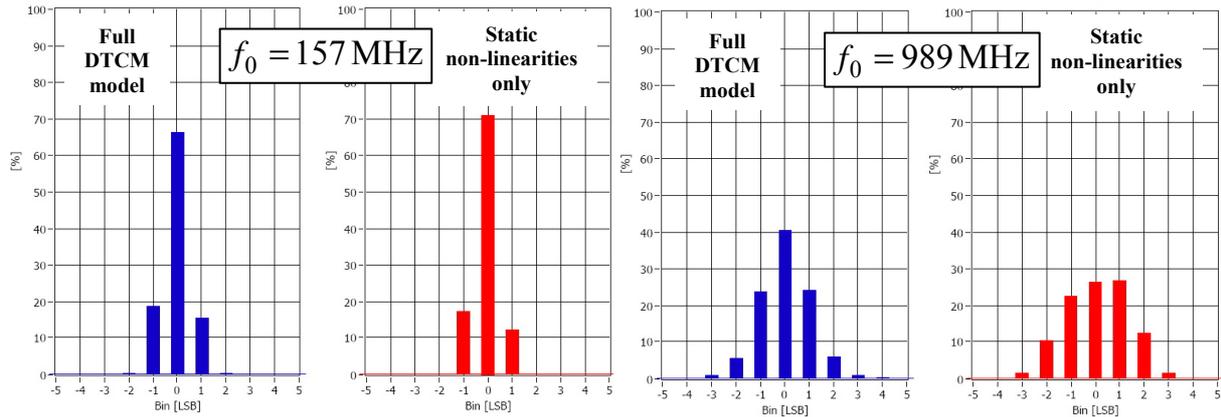
Figure 4. Statistical distributions of the deviation (unit: LSB) between the samples measured at the output of the DUT and predicted by the DTCM model, with and without the activation of the dynamic non-linearities, at low (left) and high (right) frequency (full-scale sinusoidal input at $f_0$).
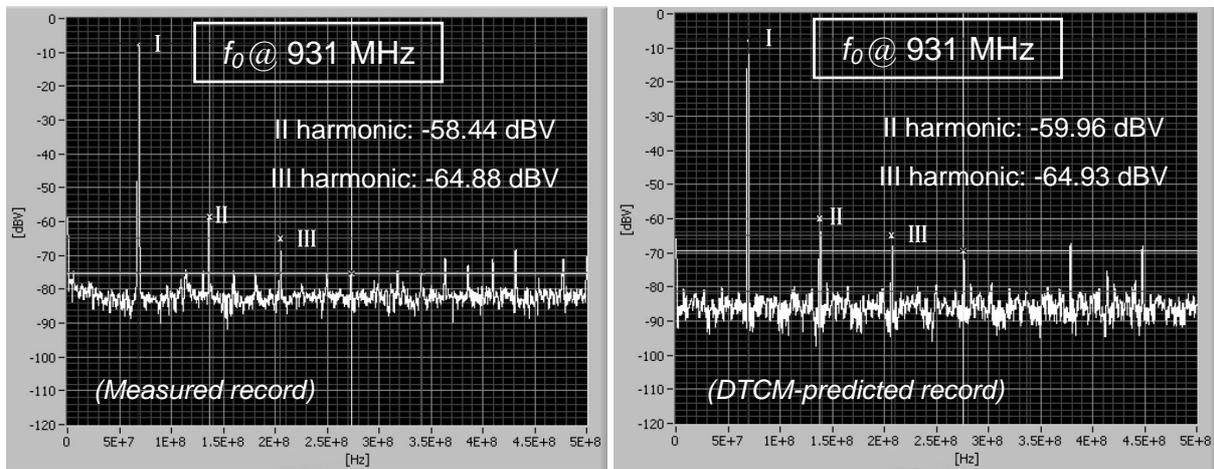


Figure 5. Comparison between the FFTs on the sample record measured at the DUT output (left) and predicted by the DTCM model (right), for a full-scale sinusoidal input at the upper-bound of the analog bandwidth.

## Acknowledgements

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