MEASUREMENT OF CYLINDER DIAMETER BASED ON SUPERPOSITION OF LASER BEAMS

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Abstract:

Nanotechnology approaching the sphere of quantum mechanics has a lot of limits connected with indirect definition of base, which should be directly related to physical constants and laws. It takes place also in measurements based on superposition of laser beams. Commonly, laser diffraction is considered to be a precise technique to measure dimensions of 3D objects, however, when required accuracy is on the level of 100nm the existing solutions do not fit to engineering applications.
T. Young interpreted the diffraction phenomena as a result of interference of the geometrical wave propagating in free space with “the edge wave”. A. Rubinowicz theoretically proved the possibility of division the Kirchhoff diffraction field into two components: incident wave and reflection wave – created by interaction between primary field and the edge of object. These mathematical considerations are commonly accepted in the scientific world, but were never proved experimentally.

In the paper the diffraction on cylindrical object is considered. Two assumptions for the theory were introduced: extensive source of light and spatial aperture in form of cylinder. The modification of the diffraction equations allowed calculating and plotting the diffraction wave with its components, and it can be considered as the first calculation and graphical representation of the real diffraction process based on Young-Rubinowicz theory.

The obtained results can be used for determination the position of the edge of an object and lead directly to the development of diameter measurement method based on strictly defined physical phenomenon.

Keywords: laser scanner, diffraction, cylinder diameter

1. INTRODUCTION

The laser measuring scanners has been proved as extremely efficient and fast tools for non-contact measurements of cylindrical objects of various size and material, such as: optical and graphite fibers, measuring wires, thin-walled tubes, cylinders, etc. Until now, the accuracy 1μm (in a small measuring range) was sufficient, but modern industry needs already the accuracy of at least one order higher.

In the most widely used high-accuracy scanners (called lasermikes), linear dimension is measured indirectly - through angle and time measuring, assuming constant scan velocity. There are many alternative solutions of laser scanners [1] enabling significant reduction of errors, but all of them face the limitation due to the difficulties in edge definition and detection. The uncertainty of diameter measurement resulting from uncertainties of the definition of the two edges of the object is undoubtedly the most difficult to determine. At this point raises the obvious analogy to the interferometric dimensional measurement of physical objects; while the displacement measurement itself can be done with nearly unlimited resolution, the determination of the beginning and end of the measured length (reference base) is subjected to considerable error. Fig.1 presents the laser-scanning-based measurement. The size and position of an object (shadow area) vary ($A_1 \rightarrow A_2$), the uncertainty of the determination of the beginning U_{Ab} and the end U_{Ue} of scanning is marked. In such measurements is often used the reference standard RS - significantly reducing an error due to the long-term scan velocity fluctuations.

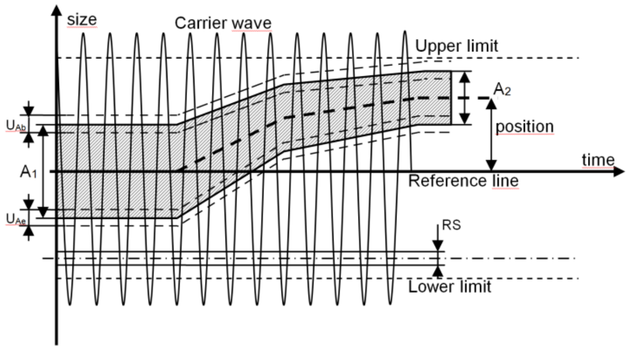

Fig. 1: Laser scanning-based measurement

It can be assumed that the contact measuring methods of cylinder diameter allow achieving the uncertainty of 0.15μm [2], while the actual position of the edge using a non-contact method is still ambiguous.

This problem is especially noticeable in nanometrology, because when approaching the sphere of quantum mechanics a number of limitations appears, associated with indirect definition of reference base (which should be directly related to physical constants and laws). This is a case also of laser diffraction-based measurement (usually considered to be a very precise technique to measure dimensions of 3D objects, and related to the exact electromagnetic formulation) that are not appropriate to deal with 3D objects. The considerations applying Fraunhofer theory are static and fragmentary, and may be concluded that the existing solutions for diffraction of 3D bodies do not fit to engineering applications.

In the paper the superposition of laser beams on cylindrical object is considered. Two assumptions for the theory were introduced: extensive source of light and spatial aperture in form of cylinder.
The definition of edge has been explicitly associated with the measurand.

2. DIFFRACTION ON SHARP EDGE AND CYLINDER

The phenomena of light diffraction and interference on the sharp edge are well described theoretically on the basis of the Huygens principle. This principle enables the creation of new phase surface behind the aperture, by drawing the envelope of the elementary waves originating around each point on the aperture.

The resulting envelope is flat only in the middle part and bends on the edges. These bends represent the rays perpendicular to the wave surface (the diffraction phenomenon). The Huygens principle does not allow to determinate the amplitude of oscillation of the waves propagating in various directions - it means that the intensity of diffracted waves is also unknown.

This problem has been solved by Fresnel, who divided the wave surface into zones – enabling to sum graphically the amplitude vectors, but in his theory, the edge of cylindrical object also is consider to be straight and sharp (radius of curvature = 0).

In Kirchhoff's theory there are no limits concerning the shape or thickness of obstruction. The influencing quantity is only the edge of obstruction itself. Following the above considerations [3] and dealing with vector potential for compound phase and amplitude structure, the arbitrary wave field can be presented as a superposition of plane waves. Since such possibility exists also for Gaussian beam, it can be assumed that diffraction on a cylindrical object is described as a decomposition of the spatial distribution of light on a series of plane waves.

In Fraunhofer diffraction, it is required that the source of light and the observation point are far from the aperture, so that the incident and diffracted wave also can be approximated by plane waves. This diffraction pattern (Fig.2) is often used in engineering applications.

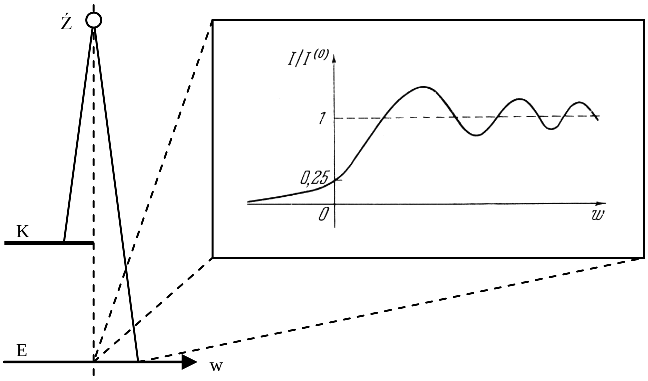

Fig.2: Fraunhofer diffraction pattern (E - observation plane, K - sharp edge)

Recently we designed a modified version of laser scanner for measurement the cross-section of bars of various form errors. When measuring the diameter, it was discovered that the measured dimension depends on the local radius of curvatures of edges. The results were confirmed by classical contact method using laser interferometer. They show that the difference in interference pattern obtained on the edges increases with the increase of the radius of curvature.

Fig.3 show the exemplary interference pattern obtained on sharp edge ≠0.1mm and polished metal cylinder Φ2mm.

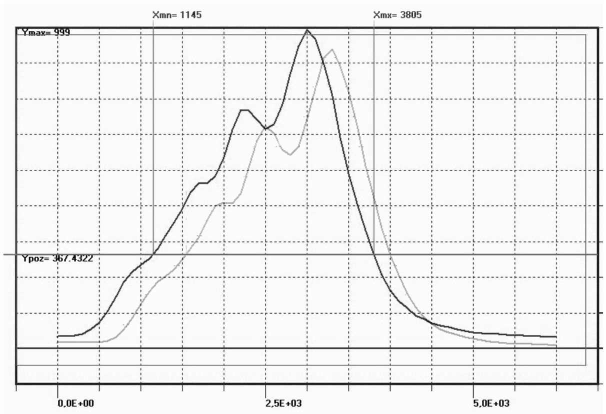

Fig. 3: Interference pattern on sharp edge ≠0,1mm (thick line) and on cylindrical object Φ2mm (thin line).

The comparison is related to intensity level 1/e. The values on abscissa are given in steps of step motor driving detector and on ordinate are presented the normalized detector signal.

The above analysis proves that the diffraction on perfect sharp edges (as in Fig.2) is only the theoretical approach and in all high accuracy engineering applications this theory should be modified [4].

3. OPTIMUM OBJECT POSITION IN RELATION TO THE LASER BEAM WAIST

The experiments performed with sharp edge and cylinder also showed the significant difference in diffraction/interference pattern when moving an object along the *z* axes in relation to beam waist [5]. On nearly sharp edge this pattern is close to theoretical one, but on cylinder it changed much. This inference is essential in all measuring instruments for the determination of measuring area and optimum location of measured object.
The experiments were limited to the area located nearby the shadow border line (edge effect) - where the strong diffraction effect is expected.

The scheme of the set-up is presented in Fig.4. An object M, polished steel cylinder, is placed at the focal distance of scan lens SL. He-Ne laser with beam expander DH and SL, all fixed to the xy-stage (Sxy), create the laser head *LH*. Scan lens forms the beam waist $2w_0$=30μm along the z axis. Using xy stage, axes of laser beam is set tangentially to the object M (displacement x_M). In each measuring position, the detector unit DT followed with aperture A=0.3mm, moves across the interference pattern (between points 1 and 2) on the long arm R_Z. Detector was calibrated with the

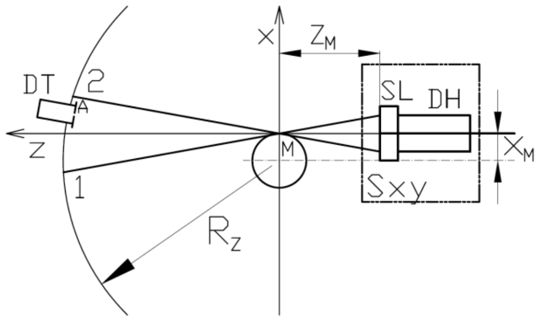

Fig.4: Scheme of experimental set-up

power meter (LaserMate-Q, Coherent) and obtained signal (in V) is proportional to the measured light intensity. Detector is driven by step motor with gear and its smallest linear displacement is 0.026μm.

Fig.4 does not show many auxiliary components like: system assuring parallel laser travel, vibration protection set-up, dark chamber, detector electronics, etc.

In experiments ϕ1.3 mm polished cylinder was used, placed perpendicularly to the laser beam axes (Fig.5). Laser beam waist is located in position B. The measurements were taken in 1mm step over 30 mm range and the measured quantity was the visibility of interference fringes (proportional to the contrast).

Fig.5 presents 4 chosen position of an object A, B, C, D (displaced every 10 mm) and Fig.6 shows the obtained results.

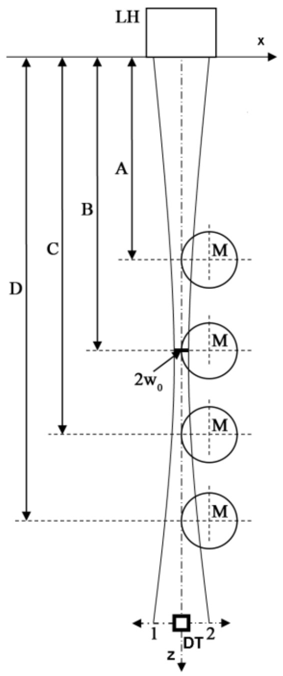

Fig.5: Object position in experiment (p.3)

Depending on the position of an object the interference pattern changes significantly. At the distance A (smaller than B) the registered pattern does not represent the diffraction distribution. The most uncertain signal was detected at the positions B close to the beam waist (it did not show any interference). With the increase of a distance the stronger and more distinct pattern appeared. The highest contrast was obtained for C. There are clearly visible 5 interference fringes and also the output signal is very strong. With further increase of a distance (D) the contrast becomes smaller.

The experiment shows that when the plane wave (at beam waist) "belts" the cylinder, the reflected wave phase varies so much that the interference patter becomes indefinite (Fig.6).

Similar conclusion comes also from Young-Rubinowicz theory which says that the resultant diffraction wave is a superposition of geometrical waves and edge waves resulting from the diffraction on hypothetical edges of the cylinder, so it is concluded that one of the main factors influencing the nature of diffraction effects, is the "belted" surface by light beam.

In further experiments an object was always located close to C position (optimum position).

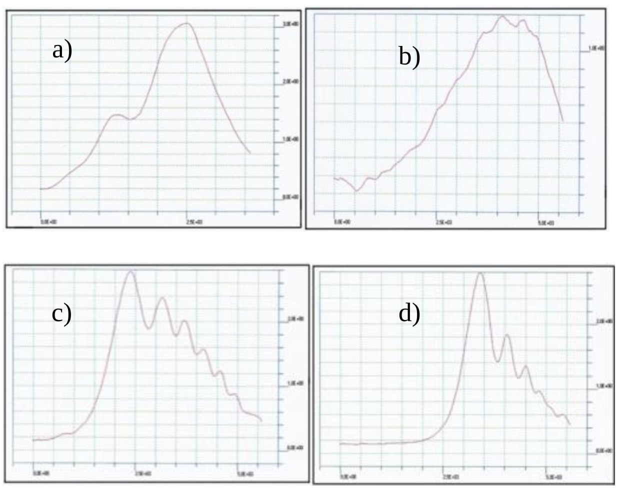

Fig.6: Output signal for chosen object positions: A, B, C, D

The performed analyses were feasible only due to the specially developed intelligent measuring system.

4. YOUNG-RUBINOWICZ EXTENDED MODEL OF DIFFRACTION

4.1.Theoretical approach

Within the Kirchhoff theory of diffraction [6] there are two possible ways of interpretation the diffraction phenomena. The first one, based on so called "edge wave", which refers to T. Young [6] idea and was later extended by A. Rubinowicz [3] and the second – Fresnel diffraction model [8], is based on the "zones concept".

The diffraction pattern described in the paper refers to the Young-Rubinowicz model [3], and is generalized to the case of diffraction on the cylindrical aperture of the light beam emitted from an extended source [9].

In the Young-Rubinowicz model the extended source is treated as a set of point sources. The resultant wave is a superposition of diffracted waves from each of the point sources, and hence is the superposition of all geometric and diffracted waves coming from all the points.

If the edge of the aperture is in the cylindrical form (Fig.7), the "belting" of the edge by light beam forms a surface area involved in the diffraction. Each generating line of cylinder is viewed as an elementary edge, hence the resultant diffraction wave is a superposition of geometrical waves and edge waves resulting from the diffraction edge on each elementary edge (k_1,..k_k) and incident waves coming from different points (x_1,…x_k) of extended source LS. It can be assumed that the larger is this area, the more component edge waves are generated and the greater is their share in the resultant diffraction wave.

It should be noted that the waves generated by the point sources will create many different shadow boundaries, among which may be distinguished three characteristic regions 1, 2, 3.

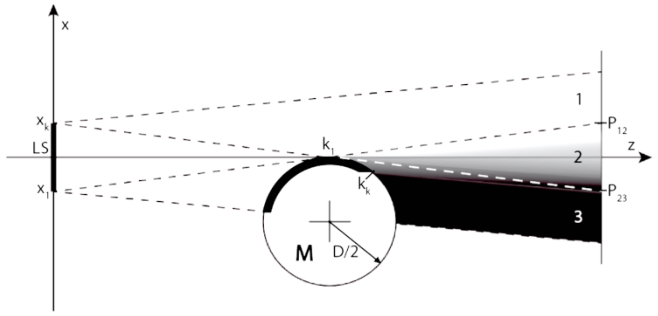

Fig. 7: Diffraction at the edge of the cylinder - the characteristic regions.

In region 1, the diffraction wave consists of geometric waves coming from all point sources. At point P_{12} the first shadow boundary exists for the extreme point x_k of point source, but this point remains fully illuminated by the sources (x_1,…x_k). In region 2 the diffraction wave is a superposition of geometrical waves and edge waves. Point P_{23} is the shadow border for the light source x_1 and is illuminated by light of the intensity level below the shadow border by all the other points of extended source (x_1,…x_k). In the region 3, the resultant diffraction wave is a superposition of edge waves from individual point sources.
Wave field originating from edge waves also occur in the area 1, however are negligible, like the fields derived from the geometrical waves in the 3 region.

Treating beam waist as extended secondary light source, on the basis of [4], it can be seen that the "belted" surface area is bigger when the beam axes is tangent to the cylinder, than when it is shifted relative thereto by a certain value.
The recorded fringe pattern is the result of interference of geometrical wave and edge wave (diffraction wave) comprising a phase-related disturbance related to a certain fragment of the measured surface; the information is enriched of the cylinder curvature.

According to the Rubinowicz model, Kirchhoff's diffraction field in an observation point (P) describes the wave motion, which is continuous everywhere in the space outside the light source and can be expressed in the form:

$$U_K(P) = U_G(P) + U_E(P) \qquad (1)$$

where:

U_G (P) - is the geometrical wave at the point of observation (it is undistorted wave, which occurs at observation point P in the absence of any aperture, if it lies on the so-called cone of light, and equals zero if the point is placed the geometric shadow area),
U_E (P) - is the edge wave at the point of observation.
In the literature, the model of the diffraction of waves proposed by Rubinowicz, and based on the Young assumptions is called the Young-Rubinowicz model and is given in details in [3].

The intensity of the resultant diffraction wave at the observation point describes the equation:

$$I_D = I_G + I_E + 2\sqrt{I_G I_E}\cos(\varphi_G + \varphi_E), \qquad (2)$$

where:
φ_G and φ_E determine the phases of the geometrical and edge waves.

To determine the intensity at each point of observation, all the components of this formula should be defined (as the functions of the observation points, according to the relations given in Fig.8).

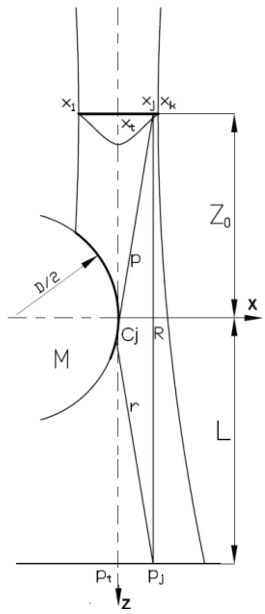

Fig.8: Geometry of the system under study

For calculating the particular waves the following notations were used:
D/2 - cylinder radius, x_1 - the starting point of the light source, x_k - end point of the light source, x_j - any point source lying in the $x_1<x<x_k$, x_t - middle source point of light lying on a tangent to the cylinder, C_j - the point of contact for x_j, z_0 - the distance between the light source and the x-axis, L - distance between the x axis and the plane of observation, ρ - segment connecting points x_j and C_j, r - segment connecting points Cj and Pj, R - segment connecting any point of source from the visible area with the point of observation.

An attempt to determine analytically the resultant edge wave and the position of the first diffraction maximum was too difficult because of the complex mathematical form of these functions. The only way to solve the above problem is to use numerical methods [9].

4.2. Numerical analysis

The geometrical wave reaches each observation point from particular unobstructed points of the laser source (of Gaussian distribution). The disturbance at any point of observation P, derived from any point of source, can be written as:

$$U_G = \frac{e^{ikR}}{R} A(P), \varphi_G = kR \qquad (3)$$

Where $A(P) = e^{-\frac{(x-x_0)^2}{\delta^2}}$ denotes the amplitude of the Gaussian beam at the point of source P, δ - is half the width of the source, x - the current point source, x_0 – the center of the source.

The resultant disturbance, from the geometrical wave, for every point of observation, can be obtained by calculating the integral of the unobstructed part of source, according to the formula (3).

The resultant disturbance from the edge wave, for each observation point, is calculated according to the formula:

$U_E = \frac{e^{i\varphi_E}}{R\sqrt{2}} A(P)$, where the phase factor is:

$e^{i\varphi_E} = C(x) + iS(x)$,

whereas the C (x) and S (x) are the Fresnel integrals:

$$C(x) = \int_0^x \cos\frac{\pi x^2}{2} dx, \quad S(x) = \int_0^x \sin\frac{\pi x^2}{2} dx$$

The parameter x according to the Rubinowicz theory [2] is determined from the formula:

$$x = \sqrt{\frac{4(r+\rho-R)}{\lambda}}$$

Using the principles of geometrical optics, defining the relative position and dimensions of the source, cylinder and observation area on the diffraction plane, and the execution of integration by Simpson method with step λ/n, where n is the number of conditioning the accuracy of the calculations, the values of geometrical and edge waves at different points of observation are obtained. The intensity of the resultant wave is given by the formula (2) where φ_G and φ_E represent the phases of geometrical and edge waves.

In accordance with the definition:

$$\left|e^{i\alpha} - e^{i\beta}\right| = \cos(\alpha - \beta),$$

in program analyzing the phenomenon of diffraction in specific observation area, it was assumed the fixed position of extended source and object.

Integration in the formulas (3) and (4) should be performed excluding shadow areas originating from the object. In calculations the independent variables are: the wavelength λ, cylinder radius D/2, geometrical dimensions of the system (z_0, L), the size of the source of Z_r, observation area (P_1 to P_2).

The center of the light source is placed on a tangent to the cylinder and perpendicular to the plane of observation. It is possible to change the position of the light source in order to analyze the effect of diffraction for different cases.

Fig.9 shows an example of the calculation for D = 2 mm, λ = 0.000633 mm, the position of the light source axes at a distance of 1 mm from the axis of the cylinder, z_0 = 100 mm, the distance from the plane of observation to axis of the cylinder L = 410mm, source diameter = 0.07 mm, the observation area from P_1 = 0.5 mm to P_2 = 3 mm.

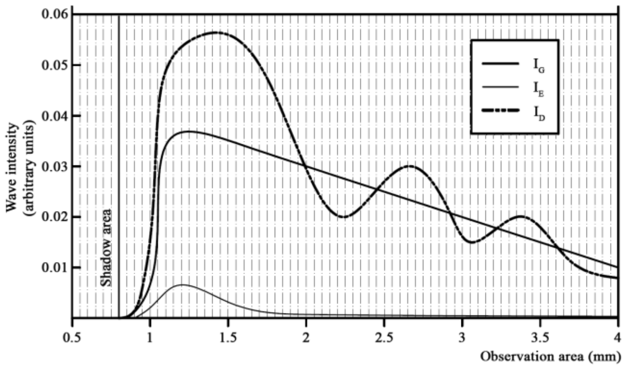

Fig.9: Courses of intensities I_G, I_E, I_D. The quantity I_D is determined from the equation (2).

In the present case a wave incidents on a cylindrical obstacle at an angle close to 0.5 π; and then the edge wave appears of reflected wave character. Occurring oscillations of diffraction wave are the result of interference of geometrical and edge waves. At observation point there is an infinite number of edge waves generated by diffraction of geometrical waves on the cylinder points considered as diffraction edges. The diffraction waves have different amplitude and incident on the particular edges at different angles. For the same point of observation the geometrical waves arrive from different points of extended source and so have different amplitude and phase. (The same conclusion drew Rubinowicz, [3]). The graphs in Fig. 8 show that the amplitudes of geometrical and edge waves indicate a clear change of their phases which causes the oscillation of diffraction wave and thus the deviation from the theory of reflection which does not take into account the phenomenon of diffraction. It can't be ruled out the presence of reflected wave with the use of extended source of the diameter r >0.1 mm. Then the phenomenon of the diffraction can be more complex. Even at an angle of incidence close to 0.5 π rad there is a clear change of phase.

The diagram shows the existence of a zero-phase of geometrical wave in the edge region of a cylindrical object. Presented waveforms demonstrate the possibility of calculation and graphical representation of the waves involved in the process of diffraction. Issue of the impact of various parameters of diffraction system on the position of diffraction curve and the location of the first diffraction maximum is crucial for optimizing the measurement system.

As a practical solution, for cylindrical aperture it is necessary to analyze the impact of the cylinder distance to the aperture and the dimensions of the source on the diffraction pattern. The position of the first diffraction maximum and its amplitude is dependent on the relative position of the source and the aperture.

5. CONCLUSION

As a result of numerical analysis of theoretical considerations extending the Young-Rubinowicz theory of diffraction it was concluded:

- Numerical calculations "in principle" confirm the theoretical conclusions of the Young-Rubinowicz diffraction theory.
- The extended model allowed the calculation and plotting diffraction, geometrical and edge waves in the diffraction process for extended source and cylindrical aperture.
- The obtained diffraction diagram allows us to determine the relationship between chosen parameters of intensity distribution and wavelength, cylinder radius, size of the light source, distance between the light source and cylinder and distance between obstacle and observation plane.

The obtained results give an answer to the very fundamental question: how to define an edge of the object being measured and how to measure it? This achievement leads directly to the development of diffraction measurement method based on strictly defined physical phenomenon. An absolute measurement of cylinder diameter requires also the determination of a distance between two opposite edges (boundary lines between smooth surfaces), but that can be measured easily with the use of laser interferometer. The expected application in laser measuring scanners (lasermikes) has been patented [10].

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