Implementation of a DSP Based Impedance Measurement Instrument Using Ellipse Fitting Algorithms

Pedro M. Ramos[1], Fernando M. Janeiro[2], Mouhaydine Tlemçani[2], A. Cruz Serra[1]

[1] *Instituto de Telecomunicações, DEEC, Instituto Superior Técnico, UTL,*
Av. Rovisco Pais 1, 1049-001 Lisbon, Portugal, e-mails: pedro.ramos@Lx.it.pt, acserra@ist.utl.pt
[2] *Instituto de Telecomunicações, Universidade de Évora, Departamento de Física,*
Rua Romão Ramalho, nº. 59, 7000-671 Évora, Portugal, e-mails: fmtj@uevora.pt, tlem@uevora.pt

Abstract- In this paper, the DSP implementation of an impedance measurement instrument based on ellipse fitting algorithms is described. The system prototype is based on a commercial DSP kit with few external electronics for baseline assessment and requirements definition. The implemented system is tested for impedance magnitudes from 100 Ω up to 15 kΩ, phases in the ±90º range at 1 kHz measuring frequency with extremely good results.

I. Introduction

New techniques based on advanced signal processing algorithms have become quite attractive for impedance measurements [1, 2]. Systems basically devoid of any analog circuitry and based on analog to digital converters (ADCs) together with sine-fitting algorithms have been successfully applied to impedance measurements with very good results [3].

Sine-fitting algorithms are used to estimate the sine signal parameters from a limited set of acquired samples. Since the ratio between the sine frequency and the sampling rate is, most of the times, not accurately known, the algorithms must also estimate the sine frequency. This makes the problem a nonlinear regression that is solved using an iterative procedure [4]. For impedance measurements the need to estimate the sine frequency and the iterative nature of the algorithm makes the use of the four-parameter sine-fitting algorithm introduced in [4] to the two channels, unadvisable. For this reason, a two-channel seven-parameter sine-fitting algorithm was introduced [5] to assure the common estimative of the sine signals frequency. However, even with specially tailored sine-fitting algorithms such as seven-parameter algorithm, their iterative nature can significantly increase the computational burden required to determine the impedance parameters, especially in noisy situations or when the initial frequency is inaccurately estimated.

A new algorithm for the estimation of the sine amplitudes, DC components and phase difference of two common frequency acquired sine signals was presented in [6]. The algorithm, based on the ellipse fitting of the two sets of samples simultaneously acquired from both channels, is non iterative and requires few calculation steps. This makes the algorithm a prime candidate for application in a DSP based impedance measurement instrument.

In this paper, the implementation of the ellipse fitting algorithm in a DSP based instrument is described. The system is used to measure a set of test impedances covering the -90 º up to 90 º phase range and impedance magnitudes from 100 Ω up to 15 kΩ at 1 kHz. These measurements demonstrate the ability of the proposed method to accurately measure impedances. The results are compared with the results obtained with the seven-parameter sine-fitting algorithm in the same measurement conditions.

II. Measurement Method

The measurement method employs a sine generator supplying current to a reference impedance connected in series with the unknown impedance. Two differential input ADCs simultaneously sample the voltage across the reference and unknown impedances. The sampled values are then transmitted to a processing unit (the DSP) in which the signal processing algorithms are implemented. The ellipse fitting algorithm estimates the ellipse parameters that best fit the XY pairs of voltages from the two channels. From the ellipse parameters, the sine amplitudes, DC components and the phase difference are determined.

With channel 1 sampling the voltage across the reference impedance and channel 2 sampling the voltage across the unknown impedance, the estimated impedance magnitude is

$$|\bar{Z}| = \frac{D_2}{D_1}|\bar{Z}_R| \qquad (1)$$

where D_i is the estimated sine amplitude of channel i and $|\bar{Z}_R|$ is the reference impedance magnitude at the measurement frequency. The estimated impedance phase is

$$\varphi_{\bar{Z}} = \Delta\phi + \varphi_{\bar{Z}_R} \qquad (2)$$

where $\Delta\phi$ is the estimated phase difference between the two acquired sine signals and $\varphi_{\bar{Z}_R}$ is the reference impedance phase at the measurement frequency.

A. Basic measurement circuit

The basic measurement circuit includes a sine generator that feeds current at the desired measurement frequency to the two impedances connected in series. However, since the algorithms estimate the ratio between the signal amplitudes and the phase difference, it is advisable that the signal amplitudes are as close as possible and also as close as possible to the ADC input range. In [3], this is achieved by the use of a set of selectable reference impedances. The algorithm performs an initial acquisition and quick estimation of the unknown impedance magnitude. With this knowledge, it selects the reference impedance that best matches the unknown impedance magnitude for the impedance measurements.
In Figure 1, the basic measurement circuit is presented. Two switches are used to select the appropriate reference impedance. The first selects the reference impedance where the sine generator current flows. The second is used to assure that the first switch's impedance is not measured together with the reference impedance. The second switch's impedance is not measured because it is in series with the voltage circuit of the four-terminal measurement setup of the reference impedance.
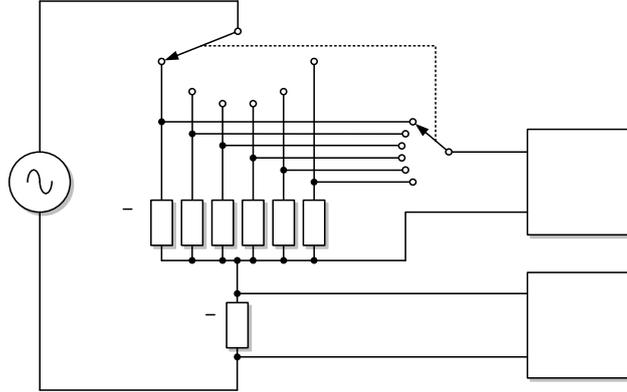

Figure 1. Basic measurement circuit. The DSP controls the two switches to maximize the ADC input voltages. The ADCs, also controlled by the DSP, sample both channels simultaneously.

B. Ellipse fitting for impedance measurements

The ellipse fitting method is based on the method first published in [7] and refined in [8]. Basically this method determines the ellipse parameters that best fit the acquired data of the two channels (u_1 and u_2). The two sine acquired signals are modeled by

$$u_1(t) = C_1 + D_1 \sin(2\pi ft + \phi_1)$$
$$u_2(t) = C_2 + D_2 \sin(2\pi ft + \phi_2) \qquad (3)$$

where C_i is the DC component of channel i. The common frequency can be eliminated from (3)

$$\left(\frac{u_1 - C_1}{D_1}\right)^2 + \left(\frac{u_2 - C_2}{D_2}\right)^2 - 2\frac{(u_1 - C_1)(u_2 - C_2)}{D_1 D_2}\cos(\Delta\phi) - \sin^2(\Delta\phi) = 0 \qquad (4)$$

with $\Delta\phi = \phi_2 - \phi_1$.

For this conic to correspond to an ellipse three conditions must be verified: $D_1 \neq 0$; $D_2 \neq 0$ and $\cos^2(\Delta\phi) < 1$, *i.e.*, (4) is either a line segment (whenever at least one of the conditions if not verified) or an ellipse. The generic conic is defined as

$$F(u_1, u_2) = au_1^2 + bu_1u_2 + cu_2^2 + du_1 + eu_2 + f = 0, \tag{5}$$

which describes an ellipse, when

$$b^2 - 4ac < 0. \tag{6}$$

By introducing a scaling constant k, this condition can be set to $4ac - b^2 = 1$.
From the ellipse parameters, the sine parameters are obtained using

$$D_1 = \frac{1}{\sqrt{ka}}, \quad D_2 = \frac{1}{\sqrt{kc}}, \tag{7}$$

$$C_1 = \frac{2cd - be}{b^2 - 4ac}, \quad C_2 = \frac{2ae - db}{b^2 - 4ac} \tag{8}$$

and

$$\cos(\Delta\phi) = -\frac{\text{sign}(k) \times b}{2\sqrt{ac}}. \tag{9}$$

Since, for impedance measurements, the actual sine amplitudes are of no interest (*i.e.*, it is the amplitude ratio D_2/D_1 that determines the impedance magnitude), there is no need to determine k. Therefore, the amplitude ratio is

$$D_2 / D_1 = \sqrt{a/c} \tag{10}$$

and the sign of the scaling constant can be replaced by the sign of parameter a (due to scaling, the parameters a and c can both be negative which would correspond also to $k < 0$).
To determine the sign of $\Delta\phi$ – unavailable from (9) – a voting system is implemented. Each sample pair $(u_{1,n}; u_{2,n})$ is used to define a complex number in the Argand plane. The sign of the phase difference between two consecutive sample pairs determines whether the ellipse is being constructed clockwise or counter clockwise. The sign of the sum of the votes (from the N acquired sample pairs) determines the sign of $\Delta\phi$. This system is specially designed for noisy and low amplitude situations.

III. DSP Implementation

The impedance measurement system was implemented in a DSP kit from Analog Devices with a Blackfin DSP (ADSP-BF533) and a multichannel Sigma-Delta ADC with 24 bit and sampling rate of 96 kS/s (AD1836). Also included in the kit, there is 64 MB of SDRAM and 2 MB of flash memory. The DSP itself has a working clock of 750 MHz, 144 kB of internal RAM and 1500 MMACS.
In Figure 2, a basic block diagram of the DSP system interface to the circuit of Figure 1 is shown.
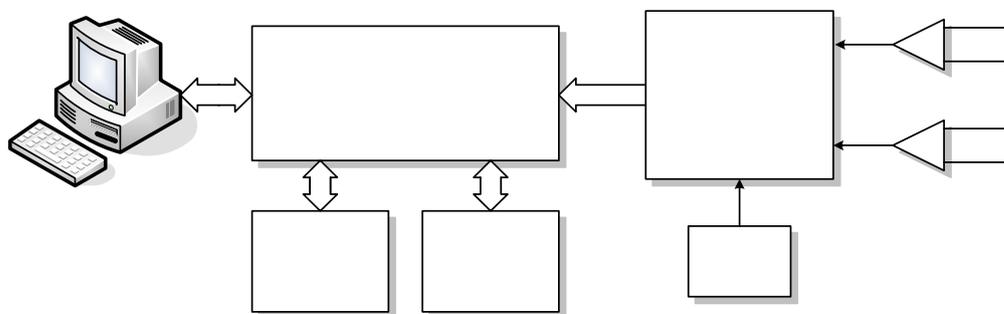

Figure 2. Basic block diagram of the DSP system that interfaces the basic electric circuit of Figure 1.

The ellipse fitting algorithm was implemented in C using the Analog Devices VisualDSP++ to program the DSP kit. The implemented subroutines include: acquisition (communication with the AD1836 acquisition IC), multiple matrix operations (*e.g.*, matrix inversion, multiplication, determination of eigenvectors and eigenvalues) and simple math operations for the estimation of the impedance magnitude and phase. A basic set of commands to interface the DSP with a PC thru a fast serial connection was also implemented. The program is stored in the non volatile flash memory and upon boot up or reset of the DSP kit, the program is loaded from the flash memory to the DSP and executed.

In addition to executing the algorithms, the DSP must also control the selection of the reference impedance. This selection usually depends on the unknown impedance magnitude. However, due to the limitations of the ellipse fitting algorithm (it cannot estimate the ellipse parameters when the signals are in phase or in opposition, *i.e.,* when $\cos^2(\Delta\phi) = 1$ because the conic is not an ellipse) the selection of the reference impedance must also be based on the unknown impedance phase. Thus, the reference impedances must include phases substantially different from 0º.

IV. Results

The implemented system was tested with the measurement of 105 different impedances with magnitudes from 100 Ω up to 15 kΩ and phases in the ±90 º range. For these impedances, a set of six reference impedances were used. In Figure 3, the measured impedances are shown together with the strategy used to select the reference impedances. The selection of the proper reference impedance is done as follows: (i) one period measurement of the two sine signals with a resistive reference impedance (*e.g.*, 1 kΩ); (ii) preliminary estimation of the impedance magnitude and phase (*e.g.*, with the non-iterative three-parameter sine-fitting used twice); if the absolute value of estimated phase is above 10º, a resistive reference impedance can be used and if the estimated impedance magnitude is below 3.5 kΩ, a 1 kΩ resistor is used as reference impedance, otherwise a 5 kΩ resistor is used; (iii) if the phase is positive and the magnitude is below 3.5 kΩ, a $1000\,\mathrm{e}^{-j90°}[\Omega]$ is used while for magnitudes above 3.5 kΩ, a $5000\,\mathrm{e}^{-j90°}[\Omega]$ is selected; (iv) if the phase is negative and the magnitude is below 3.5 kΩ, a $1000\,\mathrm{e}^{j90°}[\Omega]$ reference impedance is used and if the magnitude is above 3.5 kΩ, a $5000\,\mathrm{e}^{j90°}[\Omega]$ is used instead.
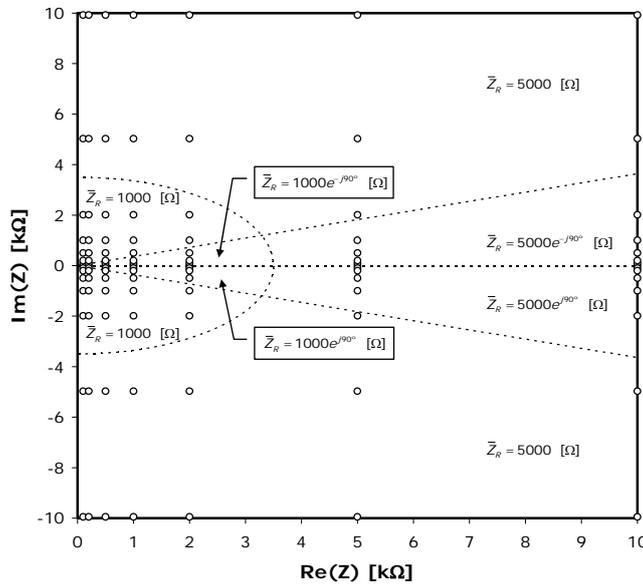

Figure 3. Set of measured impedances and the corresponding reference impedances for 1 kHz.

In Figures 4 and 6, the obtained experimental standard deviations of the estimated impedance parameters are shown as a function of the unknown impedance magnitude and phase for 1 kHz. These results were obtained with 100 repetitive measurements of each of the 105 impedances. For each measurement, 960 points per channel were acquired (this value corresponds to 10 periods of the 1 kHz sine signals). The maximum relative standard deviation of the impedance magnitude is 0.015% while the maximum standard deviation of the estimated impedance phase is 0.0082º.
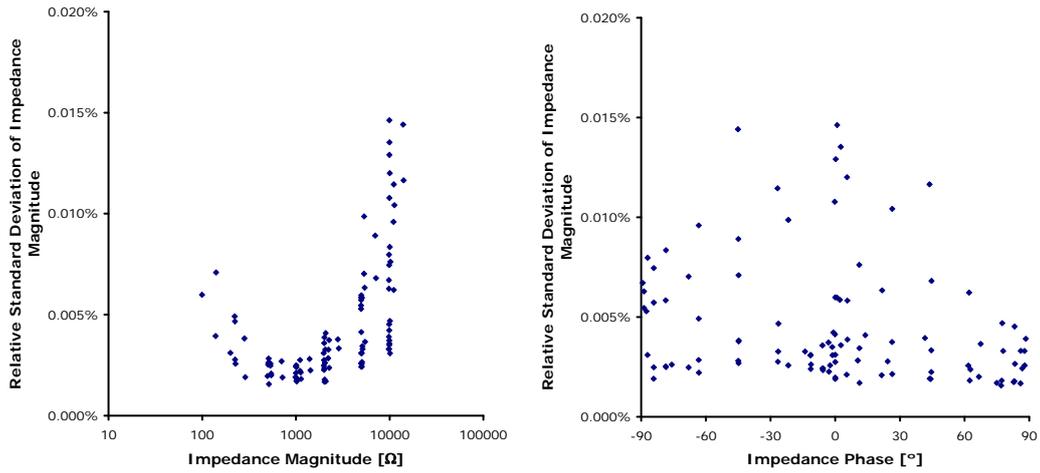
Figure 4. Relative experimental standard deviation of the estimated impedance magnitude as a function of the impedance magnitude (left) and impedance phase (right).
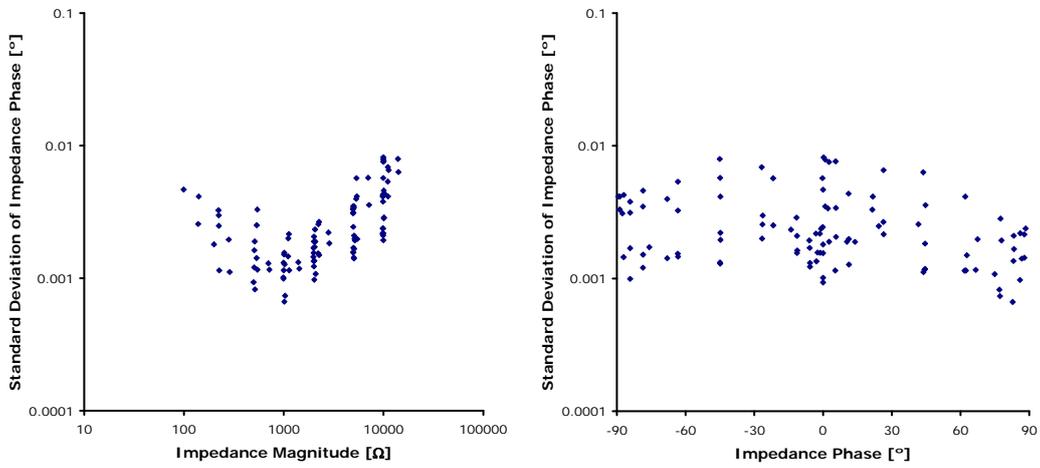

Figure 5. Experimental standard deviation of the estimated impedance phase as a function of the impedance magnitude (left) and impedance phase (right).

In Figure 6 the results obtained with the seven-parameter sine-fitting algorithm are shown for the same situations as those depicted in Figure 4 (left) and Figure 5 (left) which correspond to the ellipse fitting.
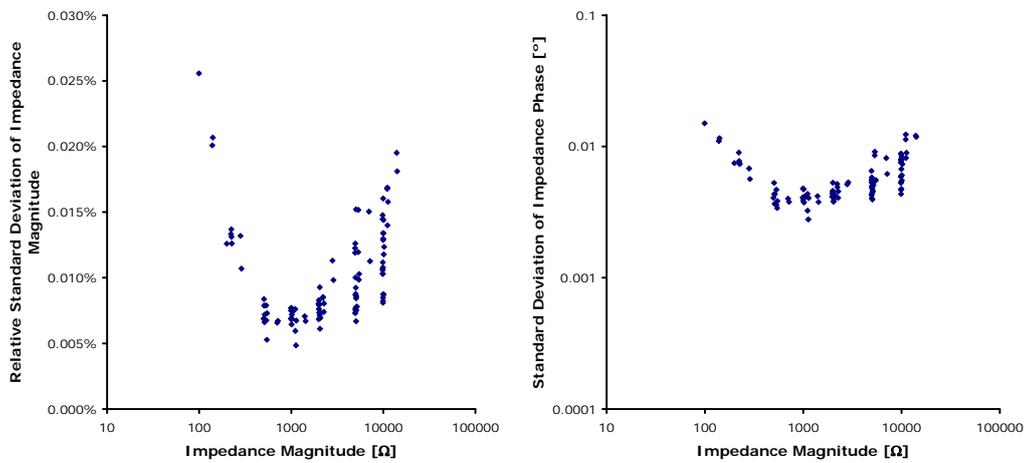

Figure 6. Experimental results obtained from the seven-parameter sine-fitting algorithm as a function of the impedance magnitude. Relative experimental standard deviation of the impedance magnitude (left) and experimental standard deviation of the impedance phase (right).

From the full set of results it can be concluded that both methods are independent of the impedance phase to be measured. Although this result was already known for the seven-parameter sine-fitting it is now also shown to be true for the ellipse-fitting algorithm – Figures 4 and 5 (right). Note that, this independence is also a result of the method used to select the reference impedance, *i.e.*, the ellipse fitting method is phase dependant because it does not work for phase differences near 0º. The algorithm results dependence on the measured impedance magnitude is as expected and is caused by the ADC resolution, the way noise affects each of the ADC channels and the low number of different reference impedances (more specifically their magnitude).

For the seven-parameter sine-fitting algorithm, the worst results correspond to a relative standard deviation of the impedance magnitude of 0.026 % and 0.015º for the experimental standard deviation of the estimated impedance phase.

Memorywise, the seven-parameter sine-fitting algorithm is considerably worse than the ellipse fitting algorithm. In the seven-parameter algorithm, a matrix with $2N$ rows and 7 columns must be constructed for each iteration and its pseudo-inverse matrix must be determined. In the ellipse fitting algorithm, due to the changes introduced in [8], all matrices are basically 3x3 which makes the required memory size practically independent on the number of acquired samples (only two vectors with the samples data must be stored).

IV. Conclusions

A DSP based impedance measurement system has been implemented. The impedance determination is based on ellipse fitting algorithms. Preliminary results are very good with relative standard deviation of the impedance magnitude below 0.015% and standard deviation of the impedance phase below 0.0082º. These results are an improvement of the results obtained with the seven-parameter sine-fitting algorithm. The system is fast in the measurement procedure with 18.84 ms per measurement (including 10 ms of acquisition time). For the seven-parameter sine-fitting algorithm and under the same conditions (*i.e.,* the same number of samples and the same impedance set), the average execution time is 523 ms and, on average, 4 iterations are required for convergence.

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