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Probabilistic evaluation of test architectures for fully differential circuits

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***Abstract*-**The paper presents probabilistic model predestined to the evaluation, comparison and optimization of test architectures for fully differential (FD) circuits. The model have the form of analytical formulae that describe the probability density functions of the magnitude and phase of the signal being measured during testing. The parameters of the models are calculated using Taylor's series approximation method. The model was validated by comparison to the results obtained by Monte Carlo simulation and applied to evaluation of test architectures for a FD bandpass filter. One of the architectures is proposed by the author. The dependence of the probabilistic features of the test responses on the type of testing circuitry and on the testing frequency is demonstrated.

I. Introduction

The slow and expensive nature of analog performance testing has motivated research into non-standard testing for analog circuits. These replacement methods are fault-driven. Analysis and simulation of analog circuits under parameter tolerances is a central issue in fault-driven testing. Applied approaches include sensitivity based estimation, vertex enumeration, deterministic and Monte Carlo sampling [1]. The most widely used approach is to perform Monte Carlo method, which is simple to apply but requires a large number of simulations. Therefore, the method is limited to small circuits that have a few uncertain parameters. Some methods that have been proposed to speed up the tolerance analysis, for example the use of interval arithmetic, often leads to overly pessimistic results.

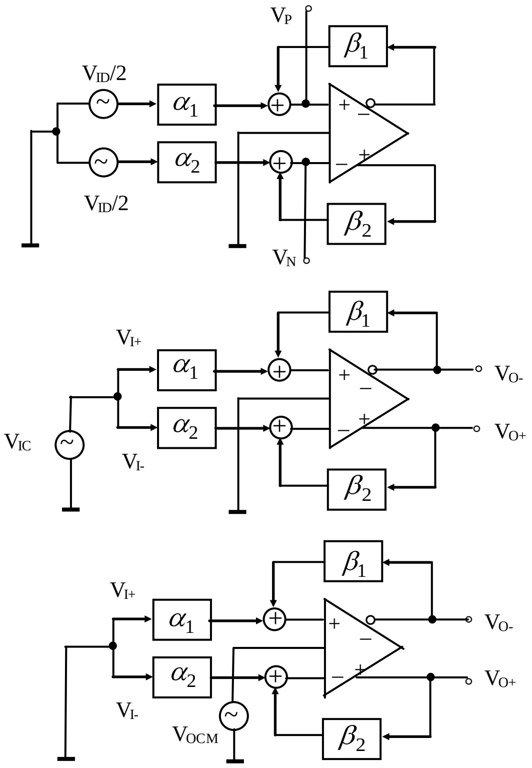
The purpose of this work is to develop a framework for the comparison, from the probabilistic point of view, of three testing circuitry for fully differential (FD) analog circuits. For problem solving, the analytical method was chosen. The solution is a probabilistic model in the form of probability density functions (pdf) of the measured signal. The parameters of the model are calculated using Taylor's series approximation method.

The paper is organized as follows. In the next section a review of test architectures for fully differential circuit is given. In Section III we derive the new distributions for the measured signal in a fault-free and faulty circuit under test (CUT). In Section IV we validate the distributions and evaluate the test architectures. Finally, some concluding remarks are given in Section V.

II. Test architectures

Fully differential circuits are increasingly common in both analog and digital electronics because differential signal processing provides many advantages, for example: the 6 dB improvement in dynamic range, increased immunity to outside electromagnetic interference, cancellation of even-order harmonics. Hence, the optimal testing methods of FD circuits are important in practice.

In the testing circuitry #1 (Fig. 1a), the CUT is excited with differential voltage V_{ID} as in [2], [3], [4]. The common-mode voltage V_{PNC} is measured at the inputs of an op-amp. The network function (1) is useful for analysis of this case. The symbols α_1, α_2, β_1, β_2 denote transfer functions of the block entering an op-amp and blocks in the feedback paths around the op-amp. In the circuitry #2, presented in Fig. 1b, the test consists of excitation of the CUT by input common-mode voltage V_{IC} and measurement of the output differential voltage V_{OD}. The network function (2) describes this configuration. The test architecture #3 is proposed by the author. The principle of test circuitry consists in measurement of the output differential voltage V_{OD} with excitation by the output common-mode voltage $V_{OC} = V_{OCM}$ (via an extra pin V_{OCM}) and both the differential inputs grounded, as shown in Fig. 1c. This configuration permits testing of the multistage FD circuits without partitioning them into separate blocks. The function (3) relates the output differential voltage to the output common-mode voltage. Ideally, due to the inherent symmetry of considered circuits, the numerators of Eqs. (1), (2), (3) are zero in a fault-free circuit. However, owing to component tolerances it cannot be guaranteed that the response of the circuit is zero in the fault-free case. Usually, a residual signal is generated. We assume that the tests are based on measurements of the magnitude of the residual signal with the aid of a comparator.



$$F_1 = \frac{V_{PNC}}{V_{ID}} = \frac{\alpha_1\beta_2 - \alpha_2\beta_1}{2(\beta_1+\beta_2)} \quad (1)$$

a)

$$F_2 = \frac{V_{OD}}{V_{IC}} = \frac{2(\alpha_1 - \alpha_2)}{\beta_1+\beta_2} \quad (2)$$

b)

$$F_3 = \frac{V_{OD}}{V_{OC}} = \frac{2(\beta_1 - \beta_2)}{\beta_1+\beta_2} \quad (3)$$

c)

Figure 1. Test architectures for fully differential CUT and respective network functions.

III. Probabilistic model

A. Derivation of the density functions for a fault-free case

The probability distributions of component parameters of the CUT are assumed to be known. Our goal is to predict the distribution of the residual signal. The central limit theorem states that a large sum of independent random variables, each with finite variance, tends to behave like a normal random variable [5]. In all considered architectures, the residual voltage is a complex random variable with the zero expected value, different variances of the in-phase and quadrature components, and correlation observed between those components. Hence, the probability density function of the residual voltage can be approximated by the joint normal pdf with three parameters (σ_X, σ_Y, r) [5]

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-r^2}}\exp\left(-\frac{1}{2(1-r^2)}\left(\frac{x^2}{\sigma_X^2} - \frac{2rxy}{\sigma_X\sigma_Y} + \frac{y^2}{\sigma_Y^2}\right)\right) \quad (4)$$

where x, y are the real and imaginary parts of voltage, σ_X, σ_Y are the respective standard deviations, r is the correlation coefficient between the x and y. The pdf of the magnitude $z = \sqrt{x^2+y^2}$ of the residual voltage, derived from (4), has the form

$$f_Z(z) = \frac{z}{\sigma_X\sigma_Y\sqrt{1-r^2}}\exp\left(-\frac{z^2}{2(1-r^2)}a\right) I_0\left(\frac{z^2}{2(1-r^2)}\sqrt{b^2+c^2}\right) \quad (5)$$

where $a = \frac{\sigma_X^2 + \sigma_Y^2}{2\sigma_X^2\sigma_Y^2}$, $b = \frac{\sigma_X^2 - \sigma_Y^2}{2\sigma_X^2\sigma_Y^2}$, $c = \frac{r}{\sigma_X\sigma_Y}$, I_0 is the modified Bessel function of the first kind and zero-th order. For r = 0 and $\sigma_X = \sigma_Y = \sigma$ the distribution (5) reduces to a very well known Rayleigh pdf

$$f_Z(z) = \frac{z}{\sigma^2} \exp\left(-\frac{z^2}{2\sigma^2}\right) \quad . \tag{6}$$

Therefore, we can call the new pdf (5) *the generalized form of the Rayleigh distribution*.

The phase $\varphi = arc\ tan\left(\frac{y}{x}\right)$ of the residual voltage has a distribution

$$f_\Phi(\varphi) = \frac{1}{2\pi} \frac{\frac{\sigma_X}{\sigma_Y}\sqrt{1-r^2}}{\cos^2\varphi - 2\frac{\sigma_X}{\sigma_Y} r \sin\varphi\cos\varphi + \left(\frac{\sigma_X}{\sigma_Y}\right)^2 \sin^2\varphi} \tag{7}$$

in the interval $(-\pi, \pi)$.

B. Derivation of the density function for faulty circuits

In a faulty circuit, X and Y are independent normal random variables with nonzero means μ_X and μ_Y respectively. It will be presented in section IV, that the zero correlation coefficient between the real and imaginary parts provides the best discrimination ability of nominal and faulty circuits in the measurement space. In the case of $r = 0$ and equal variances $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, magnitude is said to be a Rician random variable [5]. The expression for the pdf of the standard form of the Rice distribution is

$$f_Z(z) = \frac{z}{\sigma^2} \exp\left(-\frac{z^2+\mu^2}{2\sigma^2}\right) I_0\left(\frac{z\mu}{\sigma^2}\right) \tag{8}$$

where $\mu = \sqrt{\mu_X^2 + \mu_Y^2}$, $I_0(.)$ is the modified Bessel function.
An appropriate probabilistic model for magnitude responses of a faulty circuit has to represent the in-phase and quadrature components with different variances. Hence, for the faulty CUT the pdf of the magnitude, which can be called *the generalized form of the Rice distribution*, was derived as a function of four parameters $(\sigma_X, \sigma_Y, \mu_X, \mu_Y)$

$$f_z(z) = \frac{z}{2\pi\sigma_X\sigma_Y} \exp\left(-\left(\frac{z^2}{4}\left(\frac{1}{\sigma_X^2} + \frac{1}{\sigma_Y^2}\right) + \frac{1}{2}\frac{\sigma_Y^2\mu_X^2 + \sigma_X^2\mu_Y^2}{\sigma_X^2\sigma_Y^2}\right)\right)$$
$$\times \int_0^{2\pi} \exp\left(-\frac{z^2}{4}\left(\frac{1}{\sigma_X^2} - \frac{1}{\sigma_Y^2}\right)\cos 2\Theta\right) \exp\left(z\frac{\mu_X}{\sigma_X^2}\cos\Theta\right) \exp\left(z\frac{\mu_Y}{\sigma_Y^2}\sin\Theta\right) d\Theta . \tag{9}$$

The second part of the expression (9), which can not be obtained in closed form, is given by the integral but the pdf is tractable, the integral can be determined in Matlab by a process, which uses adaptive Simpson's rule.

C. Evaluation of parameters

The standard deviations σ_X, σ_Y can be evaluated performing the expansion of the network function of interest F using Taylor series expansion around the point that represents the mean values of components parameters and truncating the Taylor series after the first partial derivatives. By some rearrangements we obtain the transmission of moments formula

$$\sigma_F = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)^2 \sigma^2(x_i)} \tag{10}$$

where n is the number of variable components in the CUT, $\sigma^2(x_i)$ is the variance of the component parameter x_i.
The partial derivatives of F must be evaluated numerically

$$\frac{\partial F}{\partial x_i} \cong \frac{F(x_1^0, x_2^0, ..., x_i^0 + \Delta_i, ..., x_n^0) - F(x_1^0, x_2^0, ..., x_i^0, ..., x_n^0)}{\Delta_i} \tag{11}$$

where $F(x_1^0, x_2^0, ..., x_i^0, ..., x_n^0) = F(\mathbf{x^0})$ is the value of F evaluated at the mean component parameter values. The increments Δ_i are chosen to be equal to the standard deviations $\sigma(x_i)$ of the component values.
The correlation coefficient between the in-phase and quadrature components of voltage is by definition the ratio

$$r = \frac{\upsilon_{XY}}{\sigma_X \sigma_Y} \tag{12}$$

where υ_{XY} is the covariance between X and Y.

IV. Validation of the model and evaluation of test architectures

A. Validation of the model

The derived model can be applied for the investigation of probabilistic features of the test architectures as well as test selection, fault isolation, and fault coverage analysis. A second-order fully differential bandpass filter, shown in Fig. 5, has been chosen as the object of testing. The nominal values of parameters for R_{11}, R_{12}, R_{21}, R_{22} are [1130 Ω, 4480 Ω, 1130 Ω, 4480 Ω], all the capacitors have the same value $C_{11} \div C_{22}$ = 0.1 µF. For high precision components (tolerances from the range of 0.5% ÷ 2%) all parameters were modelled by a uniform distribution centred at their nominal values. For low precision components with tolerances from the range of 3% ÷ 5%, the normal distribution was used. The parameters of the probabilistic models for fault-free circuits with different tolerances, estimated with the aid of a first-order Taylor series approximation, are shown in Tab. 1.

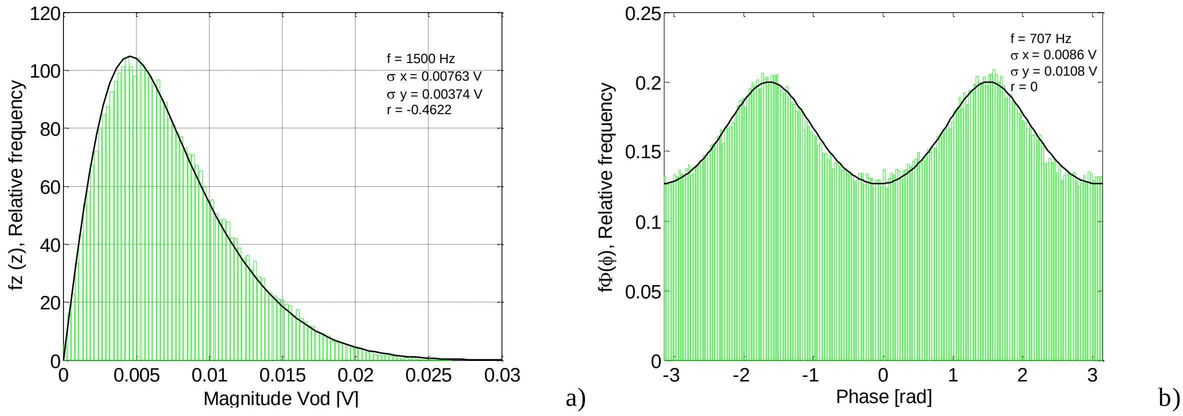


Figure 2. Comparison of the generalized Rayleigh distribution with the Monte Carlo results. A probability density function and histogram of the magnitude (a), a pdf and histogram of the phase (b)

Table 1. Parameters of the models for a fault-free CUT (f = 707 Hz)

Distribution	Uniform			Normal		
Tolerance [%]	0.5	1	2	3	4	5
σ_X [mV]	4.3	8.55	17.1	14.8	19.7	24.6
σ_Y [mV]	5.4	10.77	21.5	18.6	24.8	30.9

The model was implemented in the Matlab programming environment. In the first experiment the capabilities of the model to fit to histograms obtained by Monte Carlo method was tested. In Fig. 2 the continuous curves represent the density functions for the fitted generalized Rayleigh model of a fault-free circuit with 1% tolerances, while the histograms represent data obtained by the 10^5 - trial Monte Carlo analysis. The agreement appears to be good. Figure 3 presents the contour plot of the generalized Rician distribution for the CUT with faulty capacitor (C11 +15%) and 3% tolerances of parameters, together with the contour plot of the generalized

Rayleigh distribution for a fault-free CUT. In Fig. 4 the same distributions are compared with histograms obtained from 10^6 - trial Monte Carlo simulations. The generalized Rician model is flexible, the shape of the pdf may assume a lot of different forms. For fault-free circuit the distribution equals a generalized form of the Rayleigh distribution, while for large parameter deviations it approaches a Gauss distribution.

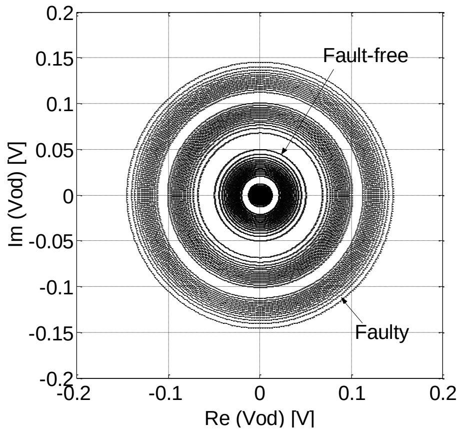


Figure 3. Contour plots of the generalized Rice distribution for a fault-free and faulty circuit

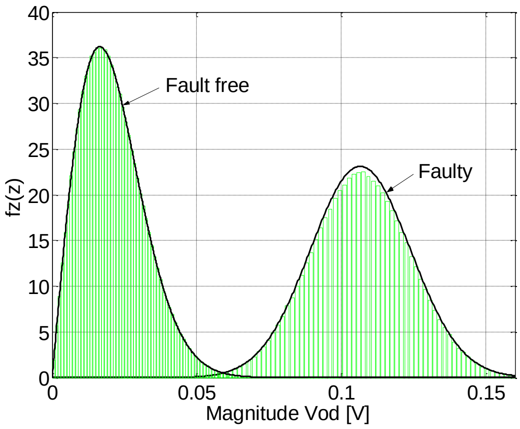


Figure 4. Comparison of the generalized Rice distribution with histograms

B. Evaluation of test architectures

The experiments illustrate the following. The parameters σ_X, σ_Y, r and consequently the shape of pdfs, depends on testing frequency. For the testing frequency equivalent to the centre frequency of the tested bandpass filter f_0, the correlation coefficient between the in-phase and quadrature components equals zero (Fig. 6). The centre frequency f_0 provides the better discrimination ability of nominal and faulty circuits in the measurement space than the non-centre one (Fig. 7).

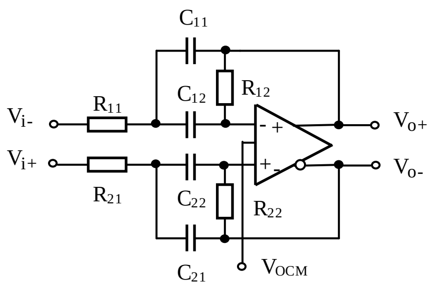


Figure 5. The second-order bandpass filter under test

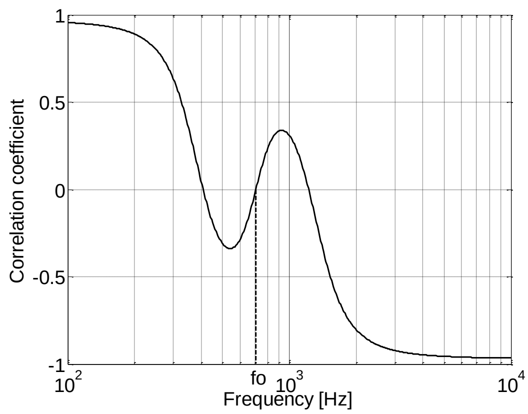
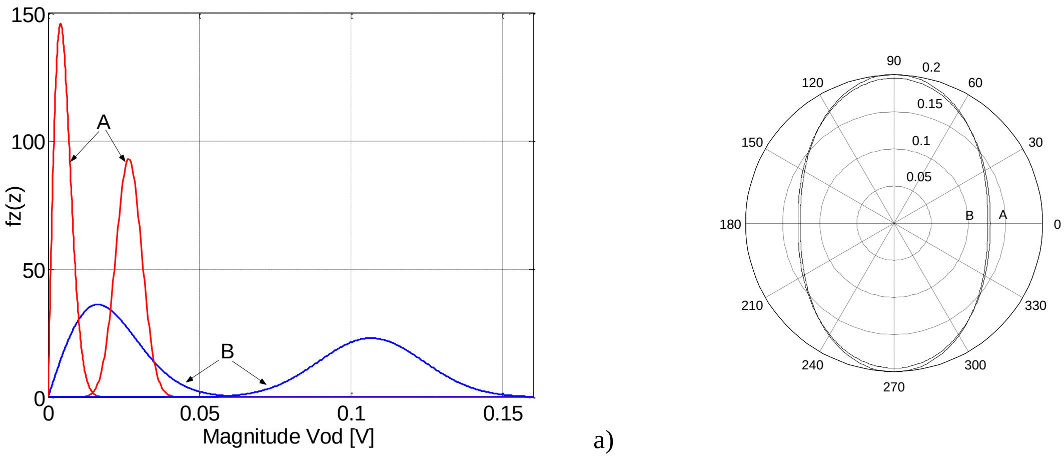


Figure 6. Correlation coefficient between the in-phase and quadrature components versus testing frequency

Interestingly, the probabilistic features of the magnitude depend on the type of testing circuitry being applied. Some examples of the distribution plots are shown in Fig. 8. It can be noted that for the circuit #1, the magnitude spreads due to tolerances (Fig. 8a, plots A) differ distinctively from magnitude spreads obtained for the circuits #2 and #3 (plots B). The differences in the density functions of the phase (Fig. 8b) are not so distinctive. The testing circuitries #2 and #3 are equivalent from the probabilistic point of view, the testing circuitry #1 differs from the others. The shapes of magnitude distributions for circuit #1 are tight and high. The steepness of the distributions causes the test quality metrics, such as yield loss and defect level, to be very sensitive to uncertainty of the comparator threshold. The circuits #2 and #3 give lower and wider distributions, which are more convenient to perform discrimination of faulty and fault-free circuits. This advantage leads to better test quality obtainable with those test architectures which exploit the common-mode excitation of the CUT.

a) b)

Figure 7. The distributions of nominal and faulty circuits projected onto the measurement space at the centre frequency of the bandpass filter under test – 707 Hz (a) and at the non-centre frequency – 300 Hz (b)



a) b)

Figure 8. The differences in the spreads of magnitude (a) and differences in the spreads of phase (b) between the testing circuitry #1 (plots A), and the testing circuitries #2 and #3 (plots B)

V. Conclusions

We have proposed model for representing probabilistic features of test responses of a fully differential circuit. The first density function is the generalized form of the Rayleigh distribution. It is relevant for representing a fault-free circuit. The second one, the generalized form of the Rician distribution, is appropriate for modelling magnitude responses of a faulty CUT as well as a fault-free circuit. Comparison between theoretical formulae and histograms, calculated from the Monte Carlo results, show good agreements. The derived model provides tools for comparing test architectures from the probabilistic point of view. It has been shown that the testing methods, which exploit common-mode excitation of the CUT, have better probabilistic features than the method, which uses the differential mode excitation. The simulation conducted on the basis of the model is extremely fast in comparison with Monte Carlo method.

References

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